# CRITERION AND FORMULA FOR CALCULATING THE FLUIDIZATION VELOCITY OF A POLYDISPERSE BED

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The conditions of transition from a real polydisperse to a fictitious monodisperse bed are examined. A formula is obtained for calculating the equivalent diameter in terms of the weight characteristics of the bed fractions. A polydispersity criterion, which takes into account the effect of granulometric nonuniformity on the fluidization velocity, is introduced. Experimental data on polydisperse materials are treated in the manner proposed. A calculation formula is derived.

In analyzing experimental data on the fluidization of polydisperse beds the formal transition to a fictitious monodisperse bed is achieved by introducing an equivalent diameter ($d_e$). The choice of formulas for calculating the equivalent diameter is determined by the starting conditions for the reduction of the polydisperse to a fictitious monodisperse bed [1–6].

Most popular is the formula

$$\frac{100}{d_e} = \sum \frac{g_i}{d_i}. \tag{1}$$

This formula was obtained for transition from a real polydisperse bed to a fictitious monodisperse bed with an equal number of particles and equal surface area. The formula has been most widely used in connection with the internal problem of fluidized-bed hydrodynamics [4, 5]; however, it is suitable only for beds of particles of the same density.

We have not encountered any published formulas for calculating the equivalent diameter of polydisperse beds consisting of particles of different density.

In relation to the external problem, it most convenient to assume that the number of particles and their volume and weight are the same for the real polydisperse and the fictitious monodisperse beds.

The starting conditions for the transition may be written as follows:

$$N = \sum_{i=1}^{k} n_i, \quad V = \sum_{i=1}^{k} v_i, \quad G = \sum_{i=1}^{k} G_i.$$

The apparent density of the particle mixture is

$$\rho_T = \sum_{i=1}^{k} \rho_i x_i.$$

The weight of the real polydisperse bed is

$$G = \sum_{i=1}^{k} n_i \frac{\pi d_e^3}{6} \rho_T. \tag{2}$$

After transforming Eq. (2) we find that the equivalent diameter of the particles of a polydisperse bed consisting of fractions differing with respect to both size and density can be calculated from the formula

$$d_e = \sqrt[3]{1 \Big/ \rho_T \left( \sum \frac{g_i}{\rho_i d_i^3} \right)}. \tag{3}$$

In order to calculate the equivalent diameter of the particles of a polydisperse mixture of materials of identical density it is possible to use the formula

$$d_e = \sqrt[3]{1 \Big/ \sum_{i=1}^{k} \frac{g_i}{d_i^3}}. \tag{4}$$

The use of formula (4), as compared with (1), intensifies the effect of the fines on the equivalent particle size. In the table, values of the equivalent diameters

Table 1

Comparative Values of Equivalent Diameters Calculated from Formula (1) and the Authors' Formula (4)

| $d_i$, mm | $g_i$ | $d_e$ acc. to (1), mm | $d_e$ acc. to (4), mm | Discrepancy, % |
|---|---|---|---|---|
| 4.37<br>5.79 | 0.507<br>0.493 | 4.98 | 4.87 | 2.2 |
| 4.37<br>5.79<br>9.85 | 0.2335<br>0.2474<br>0.5190 | 6.70 | 6.00 | 10.4 |
| 4.37<br>5.79<br>9.85<br>12.91 | 0.1655<br>0.1515<br>0.1985<br>0.4845 | 8.32 | 7.07 | 15.0 |
| 4.37<br>9.85<br>12.91<br>18.56 | 0.1280<br>0.1820<br>0.2700<br>0.4200 | 10.96 | 8.05 | 26.4 |

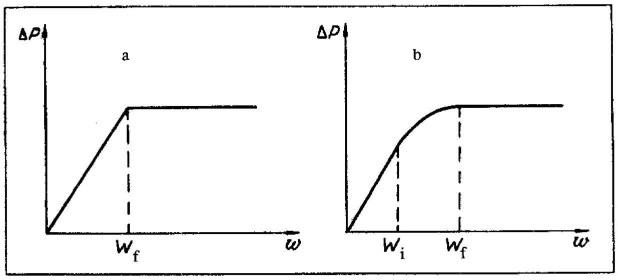


Fig. 1. Fluidization curves for monodisperse (a) and polydisperse (b) beds.

calculated from formulas (1) and (4) are compared. The discrepancy is the greater, the more complex the mixture. Although it is one of the principal parameters of a polydisperse fluidized system, the equivalent diameter cannot characterize it completely. As pointed out in [8, 9], for the complete characterization of a polydisperse system, in addition to the equivalent diameter, it is necessary to introduce a polydispersity characteristic.

We will compare the fluidization condition for mono- and polydisperse beds with the same equivalent diameter (Fig. 1). Both beds have the same equivalent diameter and the same particle number and volume. They differ not only with respect to the critical fluidization velocity but also with respect to the contact surface between the particles and the fluidizing agent. The difference in contact surfaces should be taken into account by introducing the parametric criterion

$$\frac{F_{\text{mono}}}{F_{\text{poly}}} = \frac{N d_e^2}{\sum_{i=1}^{k} n_i d_i^2}, \qquad (5)$$

$$N = \sum_{i=1}^{k} n_i,$$

$$\frac{F_{\text{mono}}}{F_{\text{poly}}} = \frac{\sum_{i=1}^{k} \dfrac{G\, g_i}{\rho_i \dfrac{\pi d_i^3}{6}} \pi d_e^2}{\sum_{i=1}^{k} \dfrac{G\, g_i}{\rho_i \dfrac{\pi d_i^3}{6}} \pi d_i^2} = \frac{d_e^2 \sum_{i=1}^{k} \dfrac{g_i}{\rho_i d_i^3}}{\sum_{i=1}^{k} \dfrac{g_i}{\rho_i d_i}}. \qquad (6)$$

The difference in the fluidization velocities of the beds must be taken into account by introducing the number $w_{\text{f poly}}/w_{\text{f mono}}$.

When a bed of monodisperse spherical particles is fluidized, the particles begin to move simultaneously. Assuming that the forces acting on a particle are proportional to the square of the diameter (surface forces) [13], for the monodisperse fraction of equivalent diameter we obtain

$$w_{\text{f mono}} \sim d_e^2.$$

For polydisperse spherical particles complete transition to the fluidized state coincides with the motion of the largest fraction. Then for the polydisperse bed, bearing in mind that collision forces are also surface forces, we obtain

$$w_{\text{f mono}} \sim d_{\max}^2$$

and

$$\frac{w_{\text{f poly}}}{w_{\text{f mono}}} \sim \frac{d_{\max}}{d_e^2}.$$

The two dimensionless numbers introduced reflect the substitution of a monodisperse for a polydisperse bed and can be grouped to obtain the dimensionless complex [10]

$$\left(\frac{F_{\text{mono}}}{F_{\text{poly}}}\right)\left(\frac{w_{\text{f poly}}}{w_{\text{f mono}}}\right) = \left(\frac{d_e^2 \sum_{i=1}^{k} \dfrac{g_i}{\rho_i d_i^3}}{\sum_{i=1}^{k} \dfrac{g_i}{\rho_i d_i}}\right)\left(\frac{d_{\max}^2}{d_e^2}\right) =$$

$$= \frac{\sum_{i=1}^{k} \dfrac{g_i}{\rho_i} \dfrac{d_{\max}^2}{d_i^3}}{\sum_{i=1}^{k} \dfrac{g_i}{\rho_i d_i}} = \frac{\dfrac{g_{\max}}{d_{\max}} \sum_{i=1}^{k} \dfrac{g_i \rho_{\max} d_{\max}^3}{g_{\max} \rho_i d_i^3}}{\dfrac{g_{\max}}{d_{\max}} \sum_{i=1}^{k} \dfrac{g_i \rho_{\max} d_{\max}}{g_{\max} \rho_i d_i}} =$$

$$= \frac{\sum_{i=1}^{k} \dfrac{g_i \rho_{\max} d_{\max}^3}{g_{\max} \rho_i d_i^3}}{\sum_{i=1}^{k} \dfrac{g_i \rho_{\max} d_{\max}}{g_{\max} \rho_i d_i}} \equiv \pi_c. \qquad (5a)$$

For a polydisperse bed of particles of identical density

$$\pi_c = \frac{\sum_{i=1}^{k} \dfrac{g_i}{g_{\max}} \dfrac{d_{\max}^3}{d_i^3}}{\sum_{i=1}^{k} \dfrac{g_i}{g_{\max}} \dfrac{d_{\max}}{d_i}}. \qquad (5b)$$

An analysis of this dimensionless complex shows that its numerator is nothing other than the number of particles in the mixture per particle of the largest fraction:

$$\sum_{i=1}^{k} \frac{g_i}{g_{\max}} \frac{d_{\max}^3}{d_i^3}.$$

After transformations the denominator can be represented as a parametric criterion of the type

$$g_{\max} d_e / d_{\max},$$

where $d_e$ is the equivalent diameter calculated from formula (1) [7]. We will assume that (5a) or (5b) is the criterion taking into account the effect of the granulo-

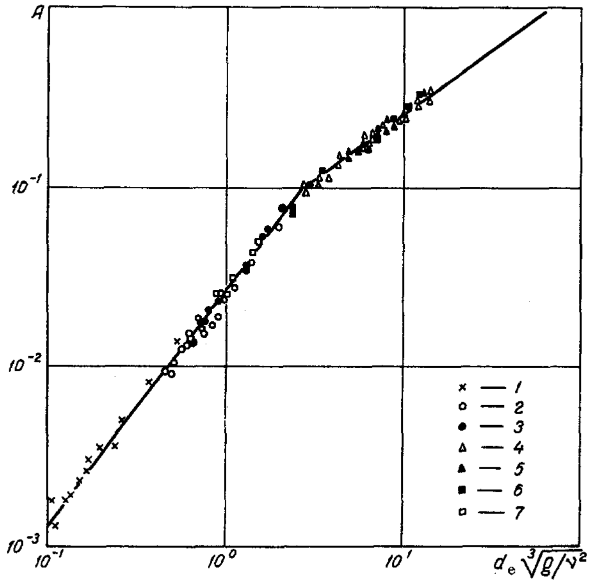


Fig. 2. Dimensionless velocity of polydisperse material in the critical state (1, 2, 3—fluidization of MSN with hydrogen, air, and carbon dioxide, respectively; 4 and 5—fluidization of silica gel and steel pellets with air; 6—fluidization of aluminosilicate with air at pressures of 1–16 atm; 7—fluidization of Khalilovsk ore with air at temperatures from 20 to 400° C).

$$A \equiv \frac{w_f}{\sqrt[3]{g\nu}}\left(\frac{\rho}{\rho_T-\rho}\right)^{0.60}\left(\frac{1}{\pi_c}\right)^{0.33}$$

metric composition on the critical fluidization velocity of the polydisperse bed.

The criterion obtained belongs to the class of criteria of parametric type with a natural scale. It is known that any dimensionless complex obtained from equations can be reduced to a parametric criterion with a physical scale of reference [10]. In a polydisperse bed the presence of geometric (granulometric) nonuniformity leads to the appearance of specific physical phenomena (progressive fluidization). The particle geometry effect can be taken into account only through a parametric criterion, since the geometric characteristic of the system is given in the bounday conditions. Thus, the criteria obtained as arguments can be divided into two groups. The first includes the criteria-complexes (they may be regarded as parametric criteria with a physical scale of reference) characteristic of both monodisperse and polydisperse beds and reflecting the hydrodynamic nature of fluidization [13]; the second group includes the parametric criteria with a natural scale of reference characteristic only of the polydisperse bed and characterizing its geometry. The proposed functional dependence

$$W = f\left(D_M;\ \frac{\Delta\rho}{\rho};\ \pi_c\right)$$

should characterize the polydisperse system more fully than the existing criterial relations [9, 11, 12].

On the basis of the experimental data on the fluidization of polydisperse beds, for which $\pi_c$ varied from 1 to 50, we found that this complex must enter into the required formula to the power 0.33.

The results of an analysis of the experimental data on polydisperse beds allowing for the effect of the granulometric composition by introducing the complex $\pi_c$ are presented in Fig. 2, which clearly shows the perfectly satisfactory approximation of our experimental data on the critical fluidization velocity of beds consisting of spherical particles (polystyrene-methylmethacrylate copolymer—MSN, silica gel, steel pellets). The corresponding formula takes the form

$$w_f = cD_M^n\left(\frac{\Delta\rho}{\rho}\right)^{0.6}\pi_c^{0.33},$$

where

$$c = 0.025 \text{ and } n = 1.3 \text{ for } D_M \leqslant 3,$$

$$c = 0.045 \text{ and } n = 0.765 \text{ for } D_M \geqslant 3.$$

NOTATION

$g_i$ is the mass fraction; $x_i$ is the volume fraction; $\rho_i$ is the density of the individual fraction; $W_f = w_f/(g\nu)^{1/3}$ is the dimensionless velocity; $D_M = d_e(g/\nu^2)^{1/3}$ is the dimensionless diameter.

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